suppression of decoherence

Seminar
Quantum problems of mesoscopic physics

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Prof. Morigi, Prof. Rieger

Susanne Blum

The paper

Dynamical suppression of decoherence in two-state quantum systems

Lorenza Viola and Seth Lloyd

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Introduction

- What is decoherence of a 'two-state quantum system'?

Decoherence is a process whereby quantum systems lose their ability to exhibit coherent behaviour such as interference.

see also Fox, Quantum Optics, chapter 13.4 Decoherence and error correction

Introduction

- Why should I be interested in suppressing decoherence in two-state quantum systems?
- need of 'storing' information

$\rightarrow$ quantum computers

Introduction

- General Idea:
  - NMR spin echo tactic
  - idea: 'reverse' time evolution
  - send series of rf-pulses

    → quantum 'bang-bang' controle

Single-qubit dephasing mechanism

- two states correspond to spins

|e> = |1>

|g> = |0>

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- System Hamiltonian:

$$H_S = \hbar \cdot \omega \cdot \frac{\sigma_z}{2}$$

Single-qubit dephasing mechanism

- Bath Hamiltonian:

$$H_B = \sum_k \hbar \cdot \omega_k \cdot b_k^\dagger \cdot b_k$$

- Interaction Hamiltonian:

$$H_{SB} = \sum_k \hbar \cdot \sigma_z \cdot \left(g_k \cdot b_k^\dagger + g_k^* \cdot b_k \right)$$

Single-qubit dephasing mechanism

- Complete Hamiltonian:

$$H = H_S + H_B + H_{SB}$$
$$= \hbar \cdot \omega \cdot \frac{\sigma_z}{2} + \sum_k \hbar \cdot \omega_k \cdot b_k^\dagger \cdot b_k + \sum_k \hbar \cdot \sigma_z \cdot \left(g_k \cdot b_k^\dagger + g_k^* \cdot b_k \right)$$

- density operator of the system

$$\rho_S(t) = Tr_B(\rho_{tot}(t))$$

Single-qubit dephasing mechanism

- Interaction picture:

$$\tilde{\rho}_{tot}(t)=e^{i(H_S+H_B)\cdot t/\hbar}\cdot\rho_{tot}(t)e^{-i\cdot(H_S+H_B)\cdot t/\hbar}$$

$$\tilde{H}(t)=\tilde{H}_{SB}(t)=\hbar\sigma_z\sum_k(g_k\cdot b_k^{\dagger}e^{i\cdot\omega_k\cdot t}+g_k^{*}\cdot b_k\cdot e^{-i\cdot\omega_k\cdot t})$$

- time evolution operator:

$$\tilde{U}_{tot}(t_{0,}t)=\exp\{\frac{\sigma_z}{2}\sum_k[b_k^{\dagger}\cdot e^{i\cdot\omega_k t_0}\xi_k(t-t_0)-b_k\cdot e^{-i\cdot\omega_k\cdot t_0}\cdot\xi_k^{*}(t-t_0)]\}$$

$$\xi_k(t-t_0=\Delta t)=\frac{2\cdot g_k}{\omega_k}\cdot(1-e^{i\cdot\omega_k\Delta t})$$

Single-qubit dephasing mechanism

- environment at thermal equilibrium at temperature T:

$$\rho_B(t_0)=\prod_k \left(1-e^{-\beta\cdot\hbar\cdot\omega_k}\right)\cdot e^{-\beta\cdot\hbar\cdot\omega_k\cdot b_k^\dagger b_k}$$

- coherence evolves to:

$$\tilde{\rho}_{01}(t)=\langle 0|Tr_B\{\tilde{U}_{tot}(t_{0,}t)\tilde{\rho}_{tot}(t_0)\tilde{U}^\dagger_{tot}(t_{0,}t)\}|1\rangle$$

$$=\tilde{\rho}_{01}(t_0)\cdot e^{-\Gamma_0(t_{0,}t)}$$

calculation

Single-qubit dephasing mechanism

- Damping factor

$$\Gamma_0(t_0, t) = \Gamma_0(t - t_0) = \frac{\sum_k |\xi_k(t - t_0)|^2}{2} \cdot \coth\left(\frac{\omega_k}{2 \cdot T}\right)$$

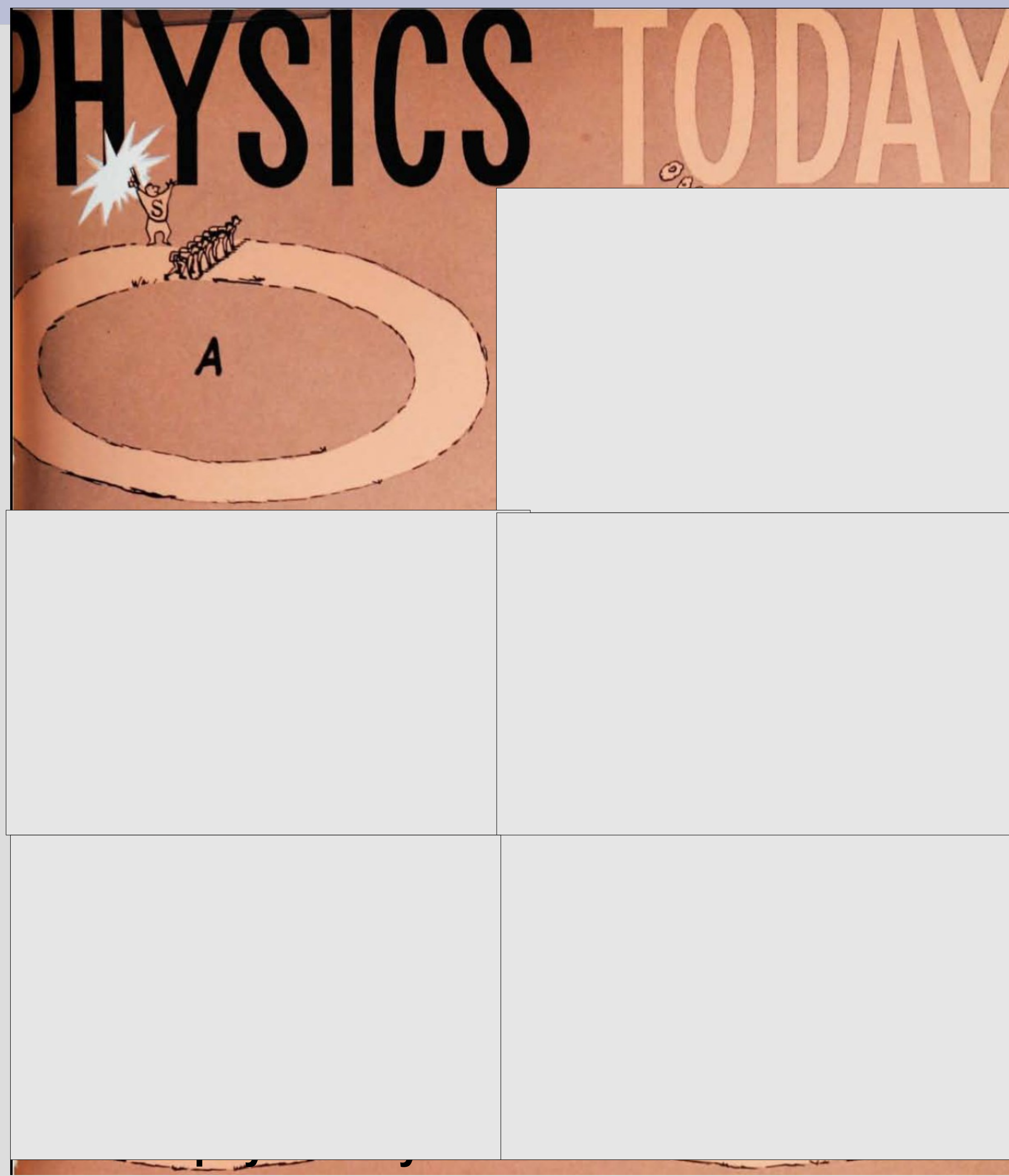
- Continuum limit:

$$\Gamma_0(t - t_0) = 4 \cdot \int_0^\infty d\omega \; I(\omega) [2 \cdot \bar{n}(\omega, T) + 1] \frac{(1 - \cos\omega \cdot (t - t_0))}{\omega^2}$$

spectral density of the bath

average number of field excitations

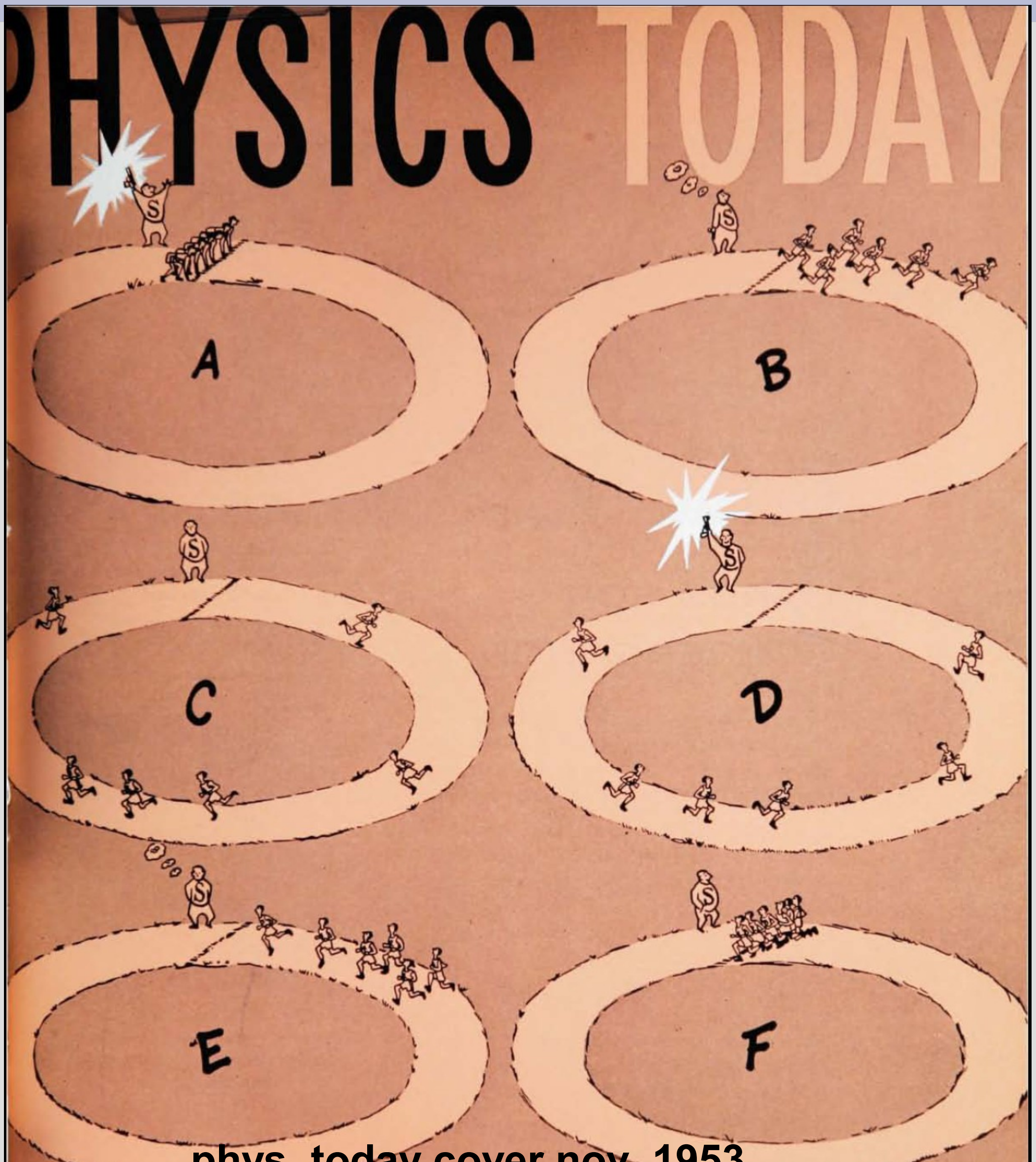
Pulsed evolution of quantum coherence



Pulsed evolution of quantum coherence

start, coherent

time evol. decoherence



phys. today cover nov. 1953

pulse, revolved evol.

stop, coherent

Pulsed evolution of quantum coherence

- Perturbation with a radio-frequency field:

$$H_{rf}(\omega_{0,}t)=\sum_{n=1}^{n_p} V^{(n)}\cdot[\cos[\omega_0(t-t_p^{(n)})]\vec{\sigma}_x+\sin[\omega_0(t-t_p^{(n)})]\cdot\vec{\sigma}_y]$$

where

$$V^{(n)}(t)=\begin{cases} V & t_p^{(n)}\leqslant t\leqslant t_p^{(n)}+\tau_p \\ 0 & elsewhere \end{cases}$$

Pulsed evolution of quantum coherence

- Evolution of coherence after N cycles

$$\tilde{\rho}_{01}(t_N) = \langle 0 | Tr_B \{ \tilde{U}_{tot}^{(N)}(t_{0,}\Delta t) \tilde{\rho}_{tot}(t_0) \tilde{U}_{tot}^{(N)\dagger}(t_{0,}\Delta t) \} | 1 \rangle$$

$$= \tilde{\rho}_{01}(t_0) \cdot e^{-\Gamma_P(N,\Delta t)}$$

Pulsed evolution of quantum coherence

- comparison between perturbed and non-perturbed coherences:

$$\Gamma_0(t_{0,}t)=\Gamma_0(t-t_0)=\frac{\sum_k |\xi_k(t-t_0)|^2}{2}\cdot\coth\left(\frac{\omega_k}{2\cdot T}\right)$$

$$\Gamma_P(N,\Delta t)=\frac{\sum_k |\eta_k(N,\Delta t)|^2}{2}\cdot\coth\left(\frac{\omega_k}{2\cdot T}\right)$$

$$\eta_k(N,\Delta t)=\xi_k(\Delta t)(e^{i\omega_k\Delta t}-1)\sum_{n=1}^{N} e^{2\mathrm{i}(n-1)\omega_k\Delta t}$$

Pulsed evolution of quantum coherence

- comparison between perturbed and non-perturbed coherences:

$$|\eta(N,\omega\Delta t)|^2 \leqslant |\xi(N,\omega\Delta t)|^2 \qquad on\,\omega\Delta t \in [0,\frac{\pi}{2}]\ for\ any\ N$$

Decoherence can be reduced!

Pulsed evolution of quantum coherence

- limit of continuous flipping:

$$\Gamma_P(N,\Delta t)=\frac{\sum_k |\xi_k(2\mathrm{N}\Delta t)|^2}{2}\coth\left(\frac{\omega_k}{2T}\right)|1-f_k(N,\Delta t)|^2$$

calculation

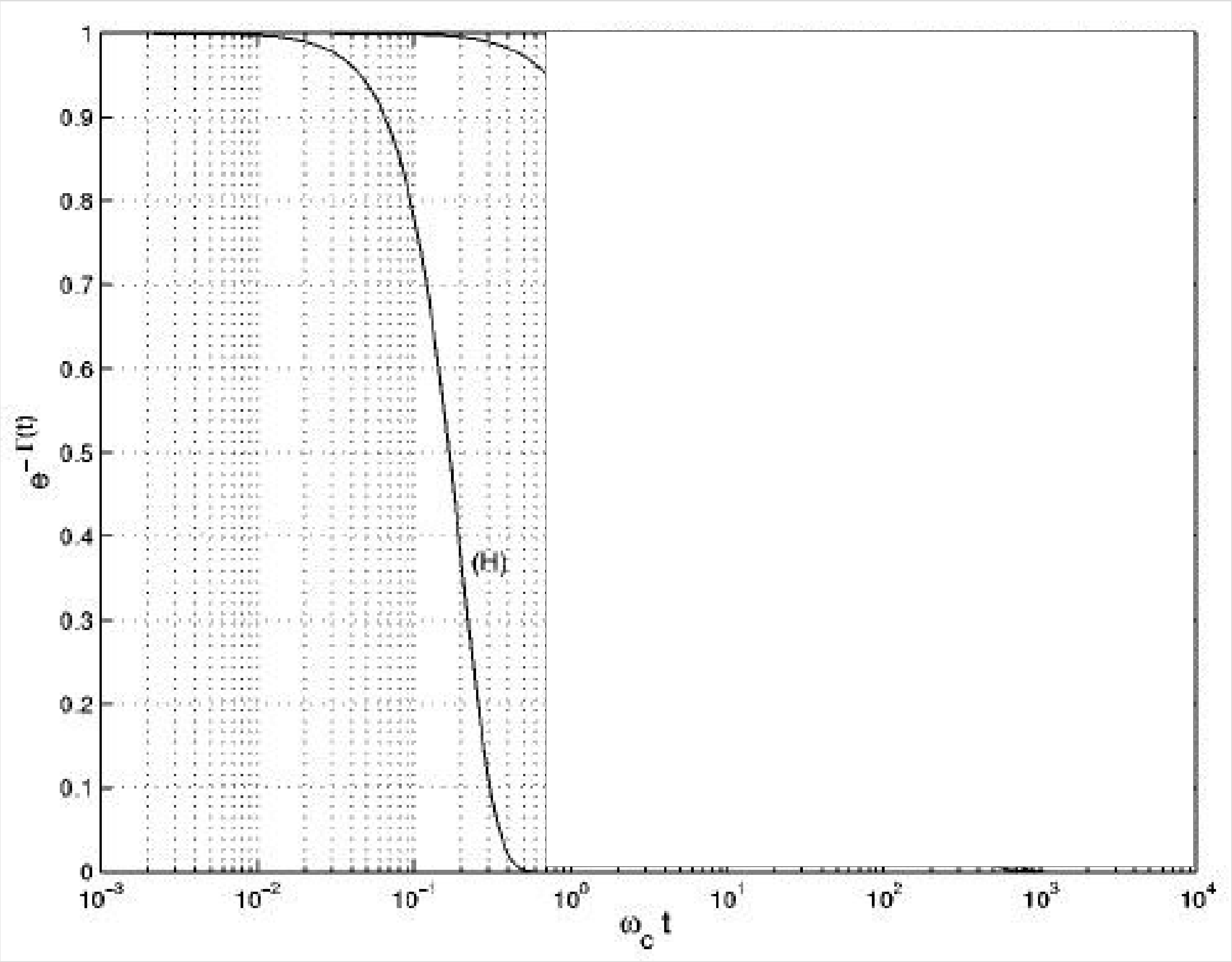
$$\lim_{\Delta t\to 0} f_k(N,\Delta t)=1$$

Decoherence vanishes!

$$f_k(N,\Delta t)=2\frac{\xi_k(\Delta t)}{\xi_k(2\mathrm{N}\Delta t)}\sum_{n=1}^{N} e^{2\mathrm{i}(n-1)\omega_k\Delta t}$$

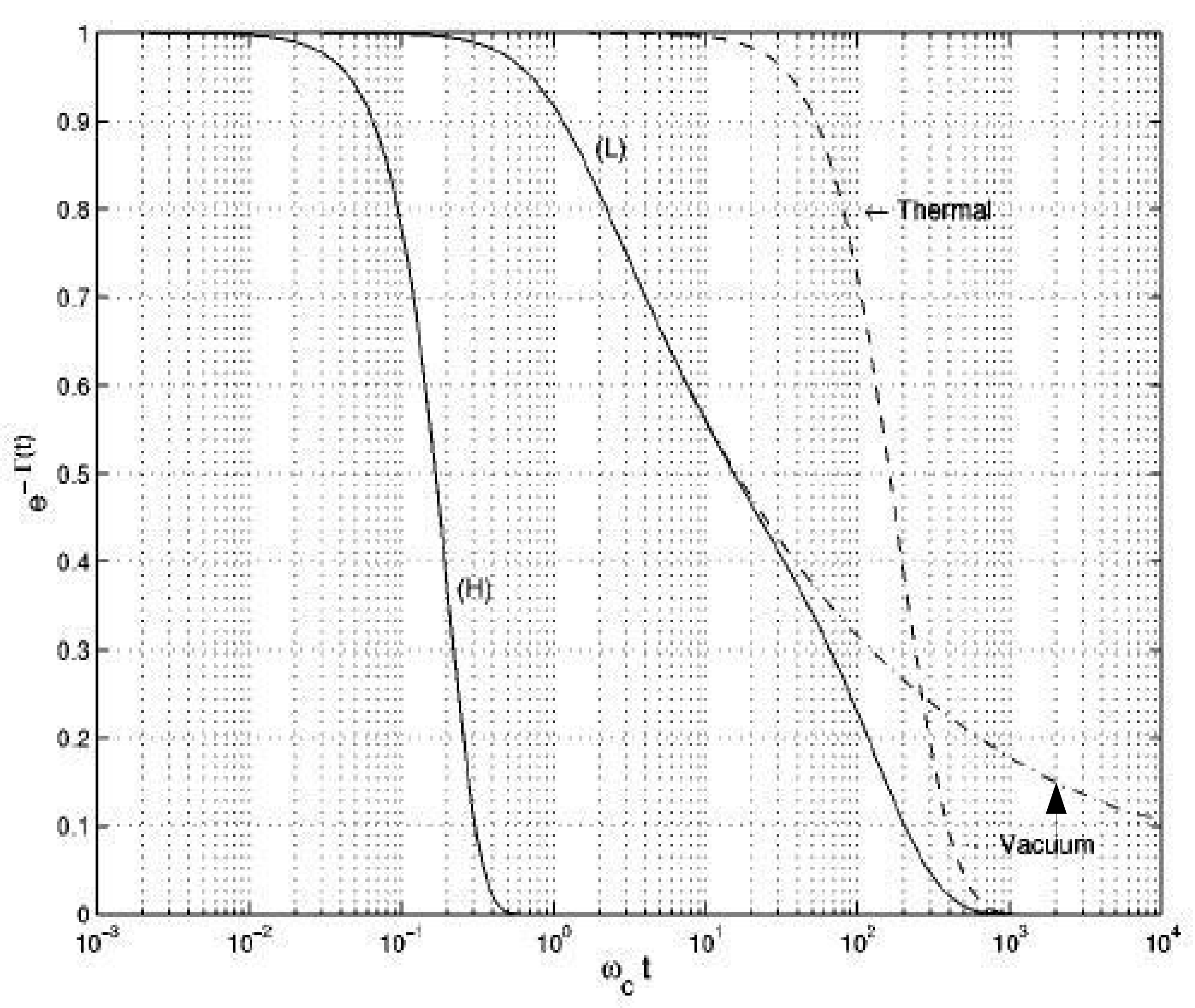
Analysis and examples

high-temperature limit (*classical* environment)



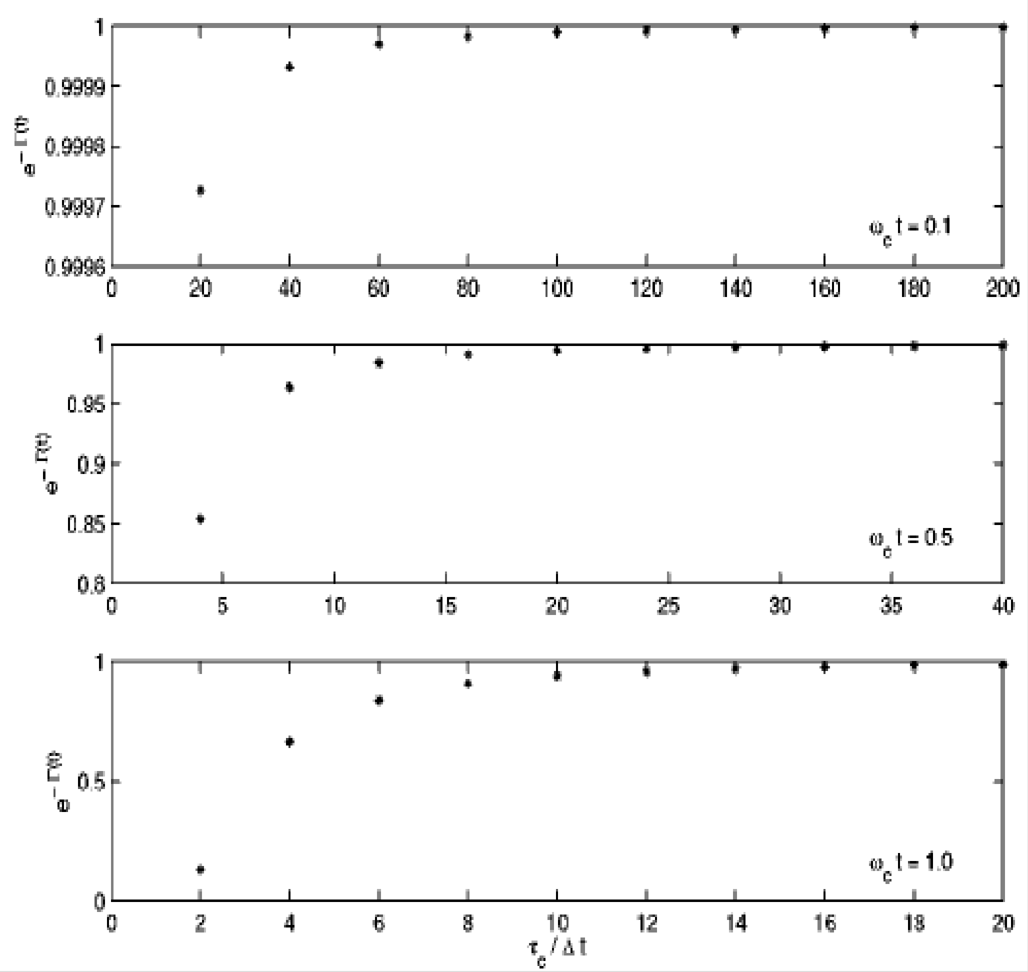
Analysis and examples

- <u>low-temperature limit (*quantum* environment)</u>



Analysis and examples

- High temperature case with 10 cycles



without flips:

$\exp\{-\Gamma\}$ =

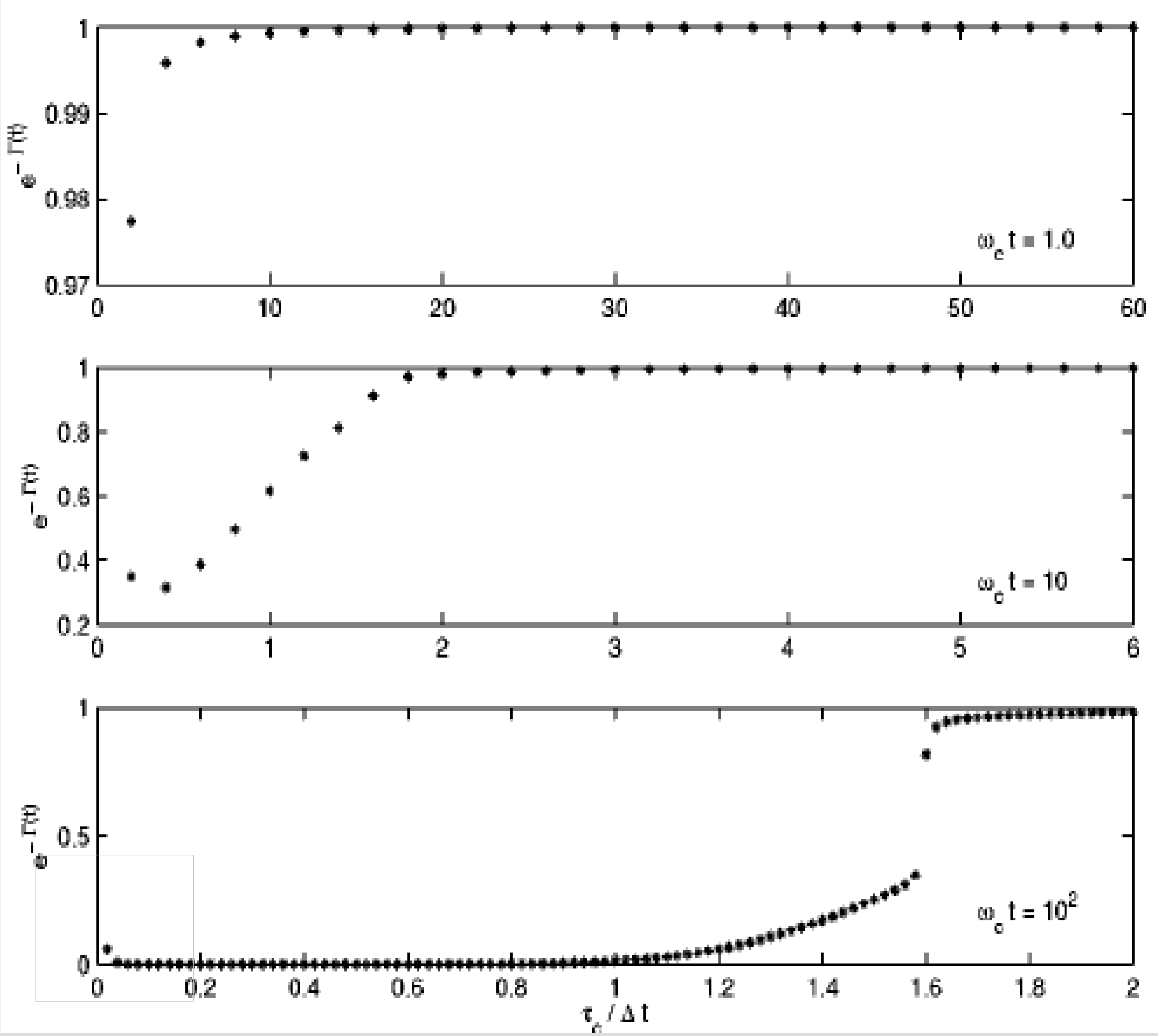
0.4

0

0

Analysis and examples

- Low temperature case with 10 cycles



without flips:
exp{-Γ} =

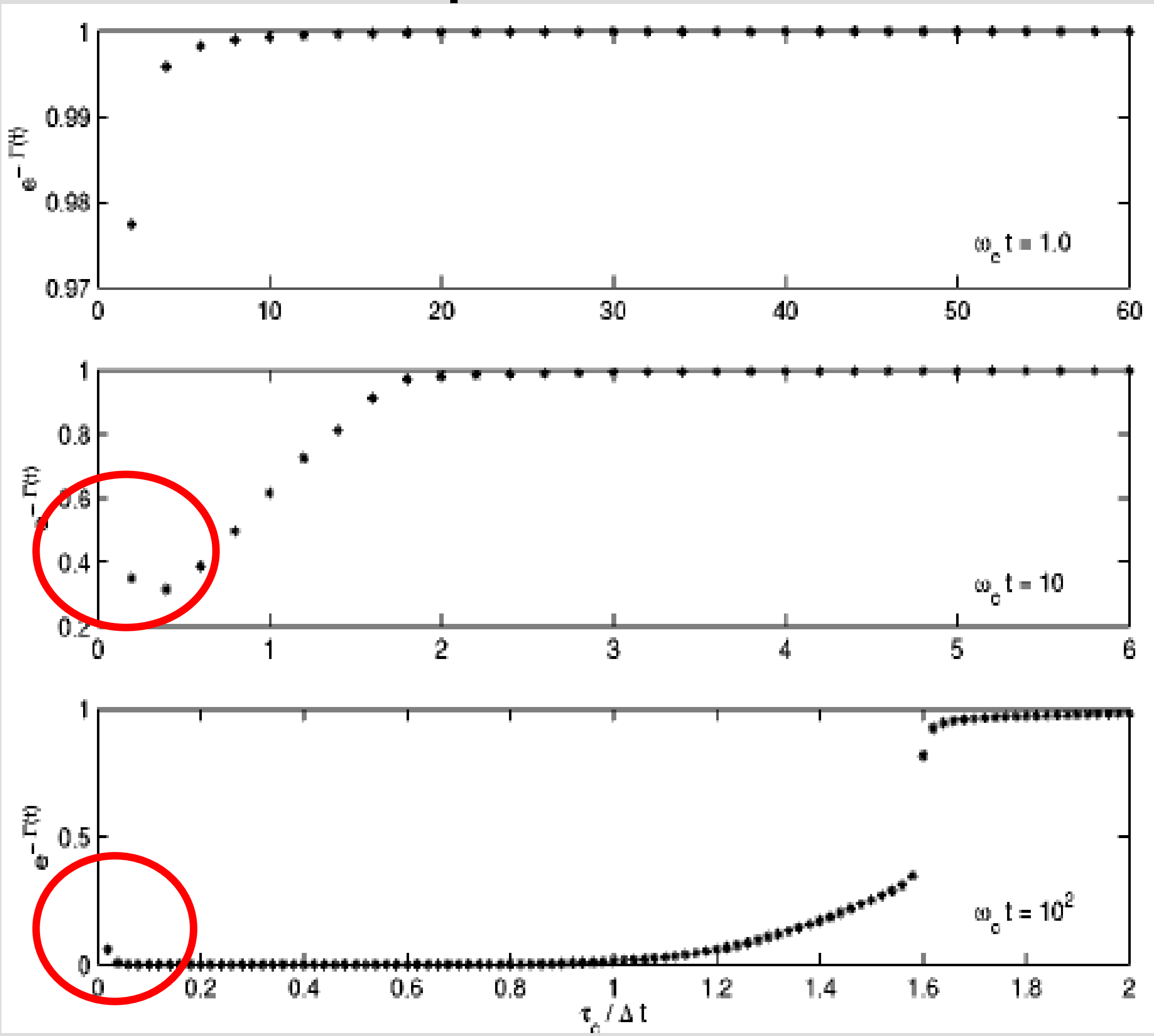
~0.915

~0.55

~0.23

Analysis and examples

- Low temperature case with 10 cycles



without flips:
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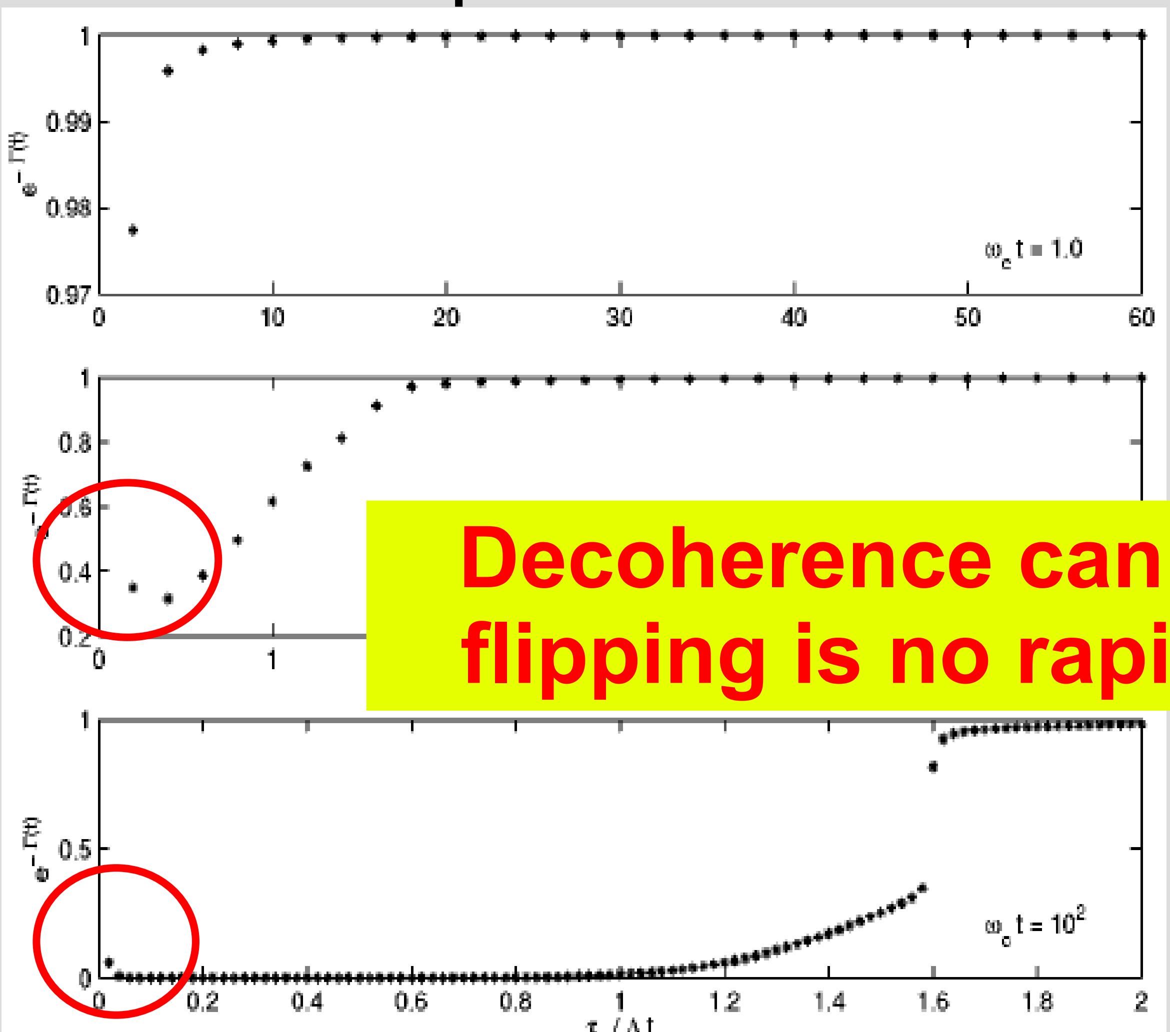
Analysis and examples

- Low temperature case with 10 cycles



without flips:
exp{-Γ} =
~0.915

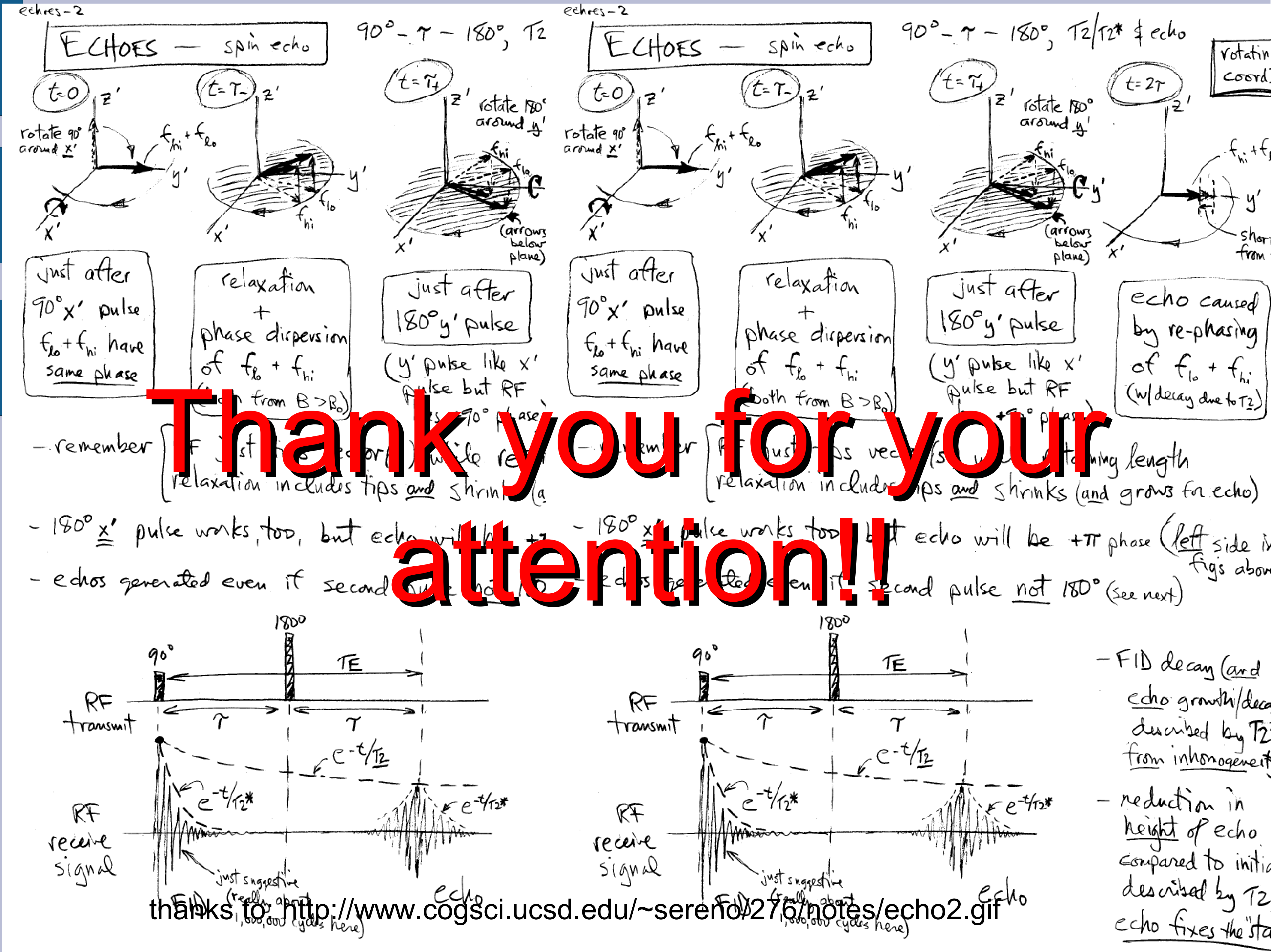
55

~0.23

Decoherence can worsen if flipping is no rapid enough!

Summary

- a state will suffer decoherence under time evolution
- decoherence can be suppressed if a pulse sequence is applied that fits the problem
- if the pulse sequence does not fit the problem, decoherence can be made worse



Thank you for your attention!!

thanks to: http://www.cogsci.ucsd.edu/~sereno/276/notes/echo2.gif

Single-qubit dephasing mechanism

back

$$\rho_{tot}(t_0)=\rho_S(t_0)\otimes\rho_B(t_0)$$

$$\tilde{\rho}_{01}(t)=\langle 0|Tr_B\{\tilde{U}_{tot}(t_{0,}t)\tilde{\rho}_{tot}(t_0)\tilde{U}^{\dagger}_{tot}(t_{0,}t)\}|1\rangle$$

$$=\tilde{\rho}_{01}(t_0)\prod_k Tr_k\{\rho_{B,k}(T)D[e^{i\cdot\omega_k\cdot t_0}\cdot\xi_k(t-t_0)]\}$$

eq.11 $$=\tilde{\rho}_{01}(t_0)\cdot e^{-\Gamma_0(t_{0,}t)}$$

with the help of:
M. Hillery *et al.*;
Phys. Rep. **106**, 121 (1984)

$$\Gamma_0(t_{0,}t)=\Gamma_0(t-t_0)=\frac{\sum_k|\xi_k(t-t_0)|^2}{2}\cdot\coth\left(\frac{\omega_k}{2\cdot T}\right)$$

$$D(\xi_k)=e^{b_k^{\dagger}\xi_k-b_k\xi_k^*}$$

Pulsed evolution of quantum coherence

$$\lim_{\Delta t\to 0} f_k(N,\Delta t)=\frac{e^{-i\omega_k t_0}}{1-e^{i\omega_k(t_N-t_0)}}\frac{\lim\limits_{\Delta t\to 0}\left(1-e^{i\omega_k\Delta t}\right)}{\Delta t}\sum_{n=1}^{N}2\Delta t\, e^{i\omega_k t_{n-1}}$$

$$=\frac{e^{-i\omega_k t_0}}{1-e^{i\omega_k(t_N-t_0)}}\frac{\lim\limits_{\Delta t\to 0}\left(1-e^{i\omega_k\Delta t}\right)}{\Delta t}\int_{t_0}^{t_N} ds\, e^{i\omega_k s}$$

$$=\lim_{\Delta t\to 0}\left[\frac{\sin\omega_k\Delta t}{\omega_k\Delta t}+i\frac{1-\cos\omega_k\Delta t}{\omega_k\Delta t}\right]=1$$

back

Ausarbeitung
eq.11 Dynamical suppression of decoherence in two-state quantum systems

Susanne Blum

23. Mai 2010

$\tilde{\rho_{01}}(t) = \tilde{\rho_{01}}(t_0)\prod_k Tr_k\{\rho_{B,k}\cdot D[e^{i\cdot\omega_k\cdot t_0}\xi_k(\Delta t)]\}$

$= \tilde{\rho_{01}}(t_0)\prod_k Tr_k\{(1-e^{-\beta\cdot\hbar\cdot\omega_k})\cdot e^{-\beta\cdot\hbar\cdot\omega_k b_k^+ b_k}\cdot e^{b_k^+\cdot e^{i\cdot\omega_k\cdot t_0}\cdot\xi_k - b_k\cdot e^{-i\cdot\omega_k\cdot t_0}\cdot\xi_k^*}\}$

$= \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot Tr_k\{e^{-2(\frac{\omega_k}{2\cdot T}b_k^+b_k}\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot e^{-b_k\cdot e^{-i\cdot\omega_k\cdot t_0}\cdot\xi_k^*}\cdot e^{b_k^+\cdot e^{i\cdot\omega_k\cdot t_0}\cdot\xi_k}\}$

$=^{[1]} \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot Tr_k\{e^{b_k^+\cdot e^{i\cdot\omega_k\cdot t_0}\cdot\xi_k}\cdot e^{-2(\frac{\omega_k}{2\cdot T})b_k^+b_k}\cdot e^{-b_k\cdot e^{-i\cdot\omega_k\cdot t_0}\cdot\xi_k^*}\}$

$= \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\pi}\int <\alpha\left|e^{b_k^+\cdot e^{i\cdot\omega_k\cdot t_0}\cdot\xi_k}\cdot e^{-2(\frac{\omega_k}{2\cdot T})b_k^+b_k}\cdot e^{-b_k\cdot e^{-i\cdot\omega_k\cdot t_0}\cdot\xi_k^*}\right|\alpha > d^2\alpha$

$= \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\pi}\int e^{\alpha^*\cdot e^{i\cdot\omega_k\cdot t_0}\cdot\xi_k-\alpha\cdot e^{-i\cdot\omega_k\cdot t_0}\cdot\xi_k^*} <\alpha\left|e^{-2(\frac{\omega_k}{2\cdot T})b_k^+b_k}\right|\alpha > d^2\alpha$

$= \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\pi}\int e^{\alpha^*\cdot e^{i\cdot\omega_k\cdot t_0}\cdot\xi_k-\alpha\cdot e^{-i\cdot\omega_k\cdot t_0}\cdot\xi_k^*}\sum_n <\alpha|e^{-2(\frac{\omega_k}{2\cdot T})b_k^+b_k}|n><n|\alpha > d^2\alpha$

$= \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\pi}\int e^{\alpha^*\cdot e^{i\cdot\omega_k\cdot t_0}\cdot\xi_k-\alpha\cdot e^{-i\cdot\omega_k\cdot t_0}\cdot\xi_k^*}\sum_n e^{-2(\frac{\omega_k}{2\cdot T})\cdot n}\frac{|\alpha|^{2\cdot n}}{n!}\cdot e^{-|\alpha|^2}d^2\alpha$

$= \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\pi}\int e^{\alpha^*\cdot e^{i\cdot\omega_k\cdot t_0}\cdot\xi_k-\alpha\cdot e^{-i\cdot\omega_k\cdot t_0}\cdot\xi_k^*}\sum_n(\frac{e^{-2(\frac{\omega_k}{2\cdot T})}\cdot|\alpha|^2}{n!})^n\cdot e^{-|\alpha|^2}d^2\alpha$

$= \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\pi}\int e^{\alpha^*\cdot e^{i\cdot\omega_k\cdot t_0}\cdot\xi_k-\alpha\cdot e^{-i\cdot\omega_k\cdot t_0}\cdot\xi_k^*}\cdot exp\{-|\alpha|^2+e^{-2(\frac{\omega_k}{2\cdot T})}\cdot|\alpha|^2\}d^2\alpha$

$= \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\pi}\int exp\{\alpha^*\cdot e^{i\cdot\omega_k\cdot t_0}\cdot\xi_k-\alpha\cdot e^{-i\cdot\omega_k\cdot t_0}\cdot\xi_k^*-|\alpha|^2\cdot(1-e^{-2(\frac{\omega_k}{2\cdot T})})\}d^2\alpha$

$=^{[2]} \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\pi}\int dr\int dk\ exp\{(r-ik)(x+iy)-(r+ik)(x-iy)-(r^2+k^2)\cdot(1-e^{-2(\frac{\omega_k}{2\cdot T})})\}$

$= \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\pi}\int dr\int dk\ exp\{\cancel{xr}+\cancel{yk}+iyr-ikx\cancel{-xr-yk}+iyr-ikx+(r^2+k^2)\cdot(-1+e^{-2(\frac{\omega_k}{2\cdot T})})\}$

$= \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\pi}\int dr\int dk\ exp\{2iry-2irx+(r^2+k^2)\cdot(-1+e^{-2(\frac{\omega_k}{2\cdot T})})\}$

$= \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\pi}\int dr\int dk\ exp\{(-1+e^{-2(\frac{\omega_k}{2\cdot T})})\{r^2+\frac{2iry}{(-1+e^{-2(\frac{\omega_k}{2\cdot T})})}+(\frac{iy}{(-1+e^{-2(\frac{\omega_k}{2\cdot T})})})^2-(\frac{iy}{(-1+e^{-2(\frac{\omega_k}{2\cdot T})})})^2+k^2-\frac{2ikx}{(-1+e^{-2(\frac{\omega_k}{2\cdot T})})}+(\frac{ix}{(-1+e^{-2(\frac{\omega_k}{2\cdot T})})})^2-(\frac{ix}{(-1+e^{-2(\frac{\omega_k}{2\cdot T})})})^2\}\}$

$= \tilde{\rho_{01}}(t_0)\prod_k(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\pi}\int dr\int dk\ exp\{\frac{y^2+x^2}{(-1+e^{-2(\frac{\omega_k}{2\cdot T})})}\}\cdot exp\{-(1-e^{-2(\frac{\omega_k}{2\cdot T})})\cdot[(r+\frac{iy}{(-1+e^{-2(\frac{\omega_k}{2\cdot T})})})^2+(k-\frac{ix}{(-1+e^{-2(\frac{\omega_k}{2\cdot T})})})^2]\}$

$= \tilde{\rho_{01}}(t_0)\prod_k\cancel{(1-e^{-2(\frac{\omega_k}{2\cdot T})})}\cdot e^{\frac{1}{2}\cdot|\xi_k|^2}\cdot\frac{1}{\cancel{\pi}}\cdot exp\{\frac{|\xi_k|^2}{(-1+e^{-2(\frac{\omega_k}{2\cdot T})})}\}\cdot\frac{\cancel{\pi}}{\cancel{(1-e^{-2(\frac{\omega_k}{2\cdot T})})}}$

$= \tilde{\rho_{01}}(t_0)\prod_k exp\{\frac{|\xi_k|^2}{2}\cdot(1+\frac{2}{(-1+e^{-2(\frac{\omega_k}{2\cdot T})})}\}$

$= \tilde{\rho_{01}}(t_0)\prod_k exp\{\frac{|\xi_k|^2}{2}\cdot coth(\frac{-\omega_k}{2\cdot T})\}$

$=^{[3]} \tilde{\rho_{01}}(t_0) exp\{\sum_k -\frac{|\xi_k|^2}{2} \cdot coth(\frac{\omega_k}{2 \cdot T})\}$

$= \tilde{\rho_{01}}(t_0) \cdot exp\{-\Gamma_0\}$

Literaturverzeichnis

[1] Tr(ABC)=Tr(CAB)=Tr(BCA)

[2] α = r +ik, $\xi_k \cdot e^{i\omega_k t_0}$=x + iy

[3] coth(-x)=-coth(x)